RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SIXTH SEMESTER EXAMINATION, JUNE 2022

THIRD YEAR [BATCH 2019-22]

Date : 20/06/2022 **PHYSICS (HONOURS)**

Time : 11 am – 1 pm **Paper : DSE 3** Full Marks : 50

Group : A

Answer **any five** questions: [5×10]

1. a) Consider a scalar function $\phi\left(x^i\right)$ in an n dimensional manifold. Show that its derivatives $\frac{\partial \phi}{\partial x^2}$ constitute a covariant tensor of rank 1. How many components does it have?

   b) An observer is accelerating with respect to an inertial observer. Acceleration a is small so that $\beta \approx 0$ throughout and the clocks of the two observers remain synchronized approximately. How should the Minkowski metric for the inertial observer transform for the accelerated observer?

   c) How is the covariant derivative of a covariant vector field defined in curved spacetime? Why should it be a tensor? The covariant derivative of a scalar is the same as its normal derivative - Explain. Find the covariant derivative of a contravariant vector. [2+4+4]

2. a) Consider a particle moving under the influence of pure gravity. How does the Principle of Equivalence allow you to introduce a freely falling frame to analyze the motion as a free particle locally. Transform to an arbitrary coordinate system x^i so that the proper time is $d\tau^2 = gijdx^i dx^j$. Find the relationship of this metric with the Minkowski one.

   b) According to the Riemannian geometry, the covariant derivative of the metric tensor $g_{ik;l} = 0$ and $\Gamma^i_{kl} = \Gamma^i_{lk}$. Using these find the relationship of the Christoffel symbols with the metric tensor and its first derivatives.

   c) What do you mean by the geodesic equation? Find the geodesic equation of motion for a test particle in a gravitational field using the concept of covariant derivative. [2+6+2]

3. In the locally inertial coordinates the curvature tensor is given by

$$R_{iklm} = \frac{1}{2}\left[g_{kl,im} - g_{il,km} + g_{im,kl} - g_{km,il}\right]$$

   a) Show that

   i) $R_{iklm} = -R_{kilm}$

   ii) $R_{iklm} = R_{lmik}$

   iii) $R_{iklm} + R_{imkl} + R_{ilmk} = 0$

   b) Find how many independent components does the curvature tensor have in a spacetime of dimension 4.

   c) Define the Ricci tensor. Show that the Ricci tensor is symmetric in its indices. [4+4+2]

4. a) Prove the Bianchi identities

$$R_{iklm;n} + R_{iknl;m} + R_{ikmn;l} = 0$$

(You can take the definition of the curvature tensor as given in Question 3)

Is this identity valid in an arbitrary coordinate system? If so, why?

b) What is the Einstein tensor? Show that the covariant derivative of the Einstein tensor is zero.

c) i) What do you mean by the energy-momentum tensor? Why do we need its covariant derivative to be zero? Give physical arguments to arrive at the Einstein equation of General Theory of Relativity without the cosmological term. What is the significance of this equation?

ii) The source of gravity is given by the energy momentum tensor of dust. Solve the Einstein equation in the limit of very weak gravity to find the Ricci tensor and the scalar curvature. [2+3+5]

5. a) What do you mean by the retrograde motion of the planets? How the ancient astronomers explain the retrograde motion of the planets? [1+2]

b) How the ancient astronomers define the magnitude scale?
(i) A star at a distance of 4 pc has an apparent magnitude 2. What is its absolute magnitude?
(ii) Given the fact that the Sun has a luminosity 3×10^{26} W and has an absolute magnitude of about 5, find the luminosity of the star. [1+1+1]

c) Determine the resolving power of Devasthal Optical Telescope (DOT) having diameter of 3.6 m at a wavelength of 5000 Å.
(i) Why this theoretical value of resolving power can't be achieved practically for ground based telescopes?
(ii) Explain briefly the different techniques which the modern astronomers developed to obtain better images even with the ground based telescopes. [1+1+2]

6. a) Derive the equation for hydrostatic equilibrium in stars and hence estimate the central pressure and central temperature of Sun using $M_{\odot} = 2 \times 10^{30}$ kg and $R_{\odot} = 7 \times 10^{8}$ m. [2+2]

b) How can you be so sure that the nuclear reactions are indeed taking place inside a star? Explain briefly the experimental set up devised by Davis to detect neutrinos. [1+2]

c) What are sunspots why do they appear on the surface of the Sun? Calculate the magnetic pressure in the center of the umbra of a large sunspot. Assume that the magnetic field strength is 0.2 T. Compare your answer with a typical value of 2×10^{4} Nm^{-2} for the gas pressure at the base of the photosphere. [2+1]

7. a) Under what conditions star formation takes place inside dark molecular cloud? Using hydrostatic equilibrium $\frac{dP}{dr} = -\frac{GM\rho}{r^2}$, determine how Jeans mass M_J and Jeans radius R_J scales with temperature and density of the molecular cloud. [2+2]

b) What are the consequences when hydrogen is exhausted inside the core of a main sequence star? [2]

c) By assuming that the conduction is the main mode of heat transfer and thermal conductivity goes a $T^{5/2}$, show that the hydrostatic equilibrium is violated in the spherical solar Corona. What conclusions you can draw from this consequence. [3+1]

8. a) What do you mean by neutron drip? Estimate the value of critical density at which electron can't stay as free particle but combine with protons to form neutrons. [**Hint:** $m_p = 1.6726 \times 10^{-27}$ kg, $m_n = 1.6749 \times 10^{-27}$, $m_e = 9.1094 \times 10^{-31}$ kg and electron number density related to Fermi momentum through the relation $n_e = \frac{8\pi}{3h^3} p_F^3$] [1+2]

b) Why do the rotating neutron stars appears to us as pulsars?
(i) The rate at which the pulsar is losing its rotational kinetic energy can be estimated from the relation

$$\frac{dK}{dt} = -\frac{4\pi^2 I \dot{P}}{P^3}$$

where I is the moment of inertia and P is its period of rotation. Estimate the kinetic energy loss rate for the Crab pulsar with $R = 10$ km, $M = 1.4\, M_{\odot}$, $P = 0.0333$ s and $\dot{P} = 4.21 \times 10^{-13}$.

(ii) Assume that all of the rotational kinetic energy lost by the pulsar is carried away by magnetic dipole radiation, $\frac{dE}{dt} = -\frac{32\pi^5 B^2 R^6 sin^2\theta}{3\mu_0 c^3 P^4}$, where B is the field strength at the magnetic pole. Estimate the magnetic field strength at the poles of the Crab pulsar assuming $\theta = 90^o$. [2+1+2]

c) A spinning neutron star of mass $M = 1.4\, M_{\odot}$ and radius $R = 10$ km. The neutron star is accreting mass from a binary companion through an accretion disk at a rate of $\dot{M} = 10^{-9}\, M_{\odot}$ per year. Determine the luminosity of the accreting object and hence its surface temperature. [2]

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